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Some Basic Concepts of Algebraic Quantum Theory

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Abstract. After briefly putting algebraic quantum theory into the context of a probabilistic interpretation with emphasis on local measurements, certain general features of the theory are examined. Sectors are defined and shown to be the components of the pure state space in the norm topology. Transition probabilities are defined by a simple algebraic formula and it is shown how superpositions of pure states may be defined. With the aid of these results, symmetries are characterized and the connexion with Wigner's Theorem is established.

1. Introduction

The predictions of quantum theory were quickly realized to be probabilistic in nature. These predictions do not fall within the scope of classical probability theory but they can be accommodated within a non-commutative probability theory. The probabilistic formulation of quantum theory owes much to the pioneering work of von Neumann [1] but the clearest account of the essentials has been provided by Mackey [2]. Here the states and the observables are treated as the primary entities of the theory and a probability measure is assigned to each pair (ω, A) consisting of a state ω and an observable A.

The algebraic approach to quantum theory with its stress on the C^*-algebra of bounded observables was initiated by Segal [3] and was realized by Araki [4], Haag and Kastler [5] to provide a useful tool for understanding local quantum field theory. The relationship of algebraic quantum theory to Mackey's axioms has recently been clarified by Plymen [6] using the concept of a Σ^*-algebra introduced by Davies [7]. As Davies showed, it is always possible to embed an abstract C^*-algebra $\mathfrak{A}$ in a canonical way in a Σ^*-algebra $\mathfrak{A}^\sim$ so that any state of $\mathfrak{A}$ has a unique extension to a σ-state on $\mathfrak{A}^\sim$. $\mathfrak{A}^\sim$ is called the σ-envelope of $\mathfrak{A}$. In classical statistical mechanics if $\mathfrak{A}$ is chosen to be the C^*-algebra of continuous functions on a compact subset of phase space, $\mathfrak{A}^\sim$ may be identified with the Σ^*-algebra of bounded Borel functions on that subset. In section 2, we discuss this relationship between C^*-algebras and Σ^*-algebras in the light of local measurements and show how unbounded observables fit naturally into the scheme.

Regarding the probabilistic formulation of quantum theory as fundamental, we take the view that a symmetry of a physical system is