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Correlation Functionals of Infinite Volume Quantum Spin Systems *

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Abstract. The existence and analyticity of the correlation functionals of a quantum lattice in the infinite volume limit is proved. The result is valid at sufficiently high temperatures and for a large class of interactions. Our method estimates the kernel K^φ for a set of Kirkwood-Salzburg equations. While a naive estimate would indicate that $\|K^\varphi\| = \infty$, we take into account cancellations between different contributions to K^φ in order to show that for sufficiently high temperatures $\|K^\varphi\| < 1$, and this estimate is independent of the volume of the system.

I. Introduction

The algebraic theory of statistical mechanics applied to quantum spin systems has recently been studied by D. ROBINSON [1, 2, 3]. In this note, it is proved that the correlation functional of an infinite volume quantum lattice satisfies a Kirkwood-Salzburg equation and is analytic in the fugacities, for sufficiently high temperatures and a large class of multiparticle potentials. This generalizes results of DOBRUSHIN [4] and GALLAVOTTI [5] for classical lattices.

In order to describe a ν-dimensional quantum lattice, assign to every point x of $\mathbb{Z}^\nu$ a Hilbert space $\mathfrak{H}_x$ of dimension N, and to every finite set $\Lambda \subset \mathbb{Z}^\nu$ the tensor product $\mathfrak{H}_\Lambda = \bigoplus_{x \in \Lambda} \mathfrak{H}_x$. The algebra of bounded operators on $\mathfrak{H}_\Lambda$, denoted $\mathfrak{A}(\Lambda)$, is called the algebra of strictly local observables, and the closure of the union $\bigcup_{\Lambda \subset \mathbb{Z}^\nu} \mathfrak{A}(\Lambda)$ is called the algebra of quasi-local observables $\mathfrak{A}$.

We will assume $N = 2$ to simplify notation, although the results are true for arbitrary N. Let the vectors $|X\rangle$, $X \subset \Lambda$, be an orthonormal basis for $\mathfrak{H}_\Lambda$. Then the algebra $\mathfrak{A}(\Lambda)$ is generated by creation and annihilation operators $a^+(X)$, $a(X)$, $X \subset \Lambda$, defined with Fermi-Dirac commutation relations at each lattice site and commutation between different lattice sites.

$$a^+(X) \equiv a^+(x_1)\, a^+(x_2) \ldots a^+(x_n)\,, \qquad X = x_1 \cup x_2 \cup \cdots \cup x_n$$
$$a^+(x_i)\, |\emptyset\rangle = |x_i\rangle$$
$$[a(x_1), a^+(x_2)]_+ = \delta_{x_1, x_2}.$$

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