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Attempt of an Axiomatic Foundation of Quantum Mechanics and More General Theories V*

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Abstract. We continue here the series of papers treated by LUDWIG in [1—5]. Using some results of DÄHN in [6], we point out that each irreducible solution of the axiomatic scheme set up in [5] is represented by a system of positive-semi-definite operator pairs of a finite-dimensional Hilbert-space over the real, complex or quaternionic numbers.

I. Introduction

Following MACKEY'S [7] general outline of axiomatic quantum theory, MACLAREN [11] and ZIERLER [8] or PIRON [12] and JAUCH [13] introduce two final axioms concerning the topological structure of the lattice G of questions (also called propositions or decision effects). This means strictly speaking that G and each sublattice of G is a compact set and that the set $A(G)$ of all atoms of G is connected. These axioms characterize the division ring appearing in the representation theorem for G.

In his axiomatic scheme (cited in [5]), LUDWIG starts from a pair of sets $(K, \hat{L})$ imbedded in a dual pair (B, B') of finite-dimensional real Banach-spaces. Hence the lattice G of decision effects, being the set of all extreme points of $\hat{L}$, carries a topological structure inherited from B'.

In [5] it was already shown that the first of the axioms mentioned above is a theorem in this exposition.

The purpose of this paper is to show that also the second axiom can be deduced. Furthermore, the following representation theorem for the system $(K, \hat{L})$ will be shown.

Theorems 20, 21. *If the dimension of the finite-dimensional Banach-spaces B, B' is large enough, then there holds:*

1. *Every irreducible solution of the axiomatic system $(K, \hat{L})$ is isomorphic to a system $(\mathscr{K}, \hat{\mathscr{L}})$ of linear operators of a finite-dimensional Hilbert-space H.*

2. *The division ring of H is isomorphic to either the real, the complex or the quaternionic number ring.*

3. *The set $\mathscr{K}$ consists of all positive-semidefinite operators V with $\boldsymbol{Tr}\, V = 1$.*

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