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Pure Operations and Measurements*

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Abstract. States of a quantum system may be influenced by an external intervention. Following HAAG and KASTLER, such a transformation of states is called an operation, and is called pure if it transforms pure states into pure states. Operations are discussed here under the assumption that they are caused by interactions with another system (apparatus), described by a S matrix. Pure operations are then shown to correspond, with one exception, to operators A with norm smaller than one. The Hermitean operators $F = A^* A$ represent quantum effects as defined axiomatically by LUDWIG. The particular case of local operations in quantum field theory is also investigated.

In algebraic quantum theory as proposed by SEGAL [1] and applied to local field theory by HAAG and KASTLER [2], observables are represented by Hermitean elements H of an abstract C^*-algebra $\mathfrak{A}$ with unit element I. States are positive linear functionals φ over $\mathfrak{A}$ with $\varphi(I) = 1$, the expectation value of H in state φ being given by $\varphi(H)$. A state φ is called pure if it cannot be decomposed as $\varphi = \alpha\varphi_1 + (1 - \alpha)\varphi_2$ with $0 < \alpha < 1$ and $\varphi_1 \neq \varphi_2$.

The (Heisenberg) state φ changes if (and only if) the system is influenced externally. In Ref. [2] interventions of this type are called operations. They are called pure operations if they transform pure states into pure states. Pure operations are assumed to be in one-to-one correspondence with elements A of $\mathfrak{A}$ with $\|A\| \leqq 1$, the pure operation corresponding to A transforming a state φ into φ_A defined by $\varphi_A(H) = \frac{\varphi(A^* H A)}{\varphi(A^* A)}$. The quantity $\varphi(A^* A)$ represents the transition probability between states φ and φ_A, and therefore φ_A is defined only if $\varphi(A^* A) \neq 0$.

Any faithful *-representation R of $\mathfrak{A}$ by a concrete C^*-algebra $R(\mathfrak{A})$ of operators acting on a Hilbert space $\mathfrak{H}$ may be used to describe the system under question in the usual Hilbert space framework of quantum mechanics, different faithful *-representations being physically equivalent [2]. Let us furthermore assume that $\mathfrak{A}$ is primitive, i.e., as possessing an irreducible faithful *-representation.

Subsequently, a fixed irreducible faithful *-representation R of $\mathfrak{A}$ will be used throughout. We will use the letters $\mathfrak{A}$ for $R(\mathfrak{A})$ and X for $R(X)$, $X \in \mathfrak{A}$, i.e., $\mathfrak{A}$ will be identified with the concrete C^*-algebra $R(\mathfrak{A})$ on

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