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On Locally Normal States in Quantum Statistical Mechanics

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Abstract. A sufficient condition is given in order that a von Neumann algebra with cyclic vector is quasi-standard. With the help of this result it is proved that a locally normal state with a cyclic and separating vector in the representation space gives rise to a quasi-standard von Neumann algebra. Furthermore it is proved that the representation space determined by a locally normal state in the G.N.S. construction is separable.

I. Introduction

The results of this work have their origin in two recent papers [4, 9]. In [9] TOMITA proves the highly non-trivial result that a von Neumann algebra which has a cyclic and separating vector is quasistandard. In this paper an easy proof of this result will be given for a special case which is of particular interest for physical applications, where the von Neumann algebra considered is generated by the representation of a C^*-algebra of quasi-local observables determined by a locally normal state.

Following [4] we define an algebra $\mathfrak{A}$ of quasi-local observables as a norm-closed algebra of operators in Fock-space $\mathfrak{H}_F$: to every finite volume V is assigned the algebra $\mathfrak{A}(V)$ of all bounded operators operating in the sub-Fock space $\mathfrak{H}_F^V \subset \mathfrak{H}_F$, hence $\mathfrak{A}(V)$ is a norm closed and weakly closed algebra; then $\mathfrak{A}$ is defined in $\mathfrak{H}_F$ as the closure in the norm topology of the union of the $\mathfrak{A}(V)$ for all finite V, $\mathfrak{A} = \overline{\bigcup_V \mathfrak{A}(V)} = \overline{\mathfrak{A}}_L$, where the "local" algebra $\mathfrak{A}_L$ is defined by $\mathfrak{A}_L = \bigcup_V \mathfrak{A}(V)$.

In [4] the G.N.S. representation determined by a normal state

$$\omega_V(A) = \mathrm{Tr}_V \varrho_V A,\ A \in \mathfrak{A}(V)$$

defined over $\mathfrak{A}(V)$, with the additional property $\omega_V(A^*A) = 0$ implies $A = 0$ for every $A \in \mathfrak{A}(V)$ is constructed. Here ϱ_V is a density operator in $\mathfrak{H}_F^V$ with $\mathrm{Tr}_V \varrho_V = 1$ (the index V in Tr_V denotes that the trace is taken in the Hilbert space $\mathfrak{H}_F^V$). One of the results that can be inferred from that paper is that the von Neumann algebra generated by the representatives of the elements from $\mathfrak{A}(V)$ is quasi-standard [5], that means is the left representation of a certain quasi-unitary algebra $\tilde{\mathfrak{A}}(V) \subset \mathfrak{A}(V)$. (For a fuller discussion of quasi-standard von Neumann