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The N-Body Problem with Spin-Orbit or Coulomb Interactions

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Abstract. A study is made of the resolvent $R(\lambda)$ for a system of n particles with spin-orbit coupling, an interaction which necessarily has a long range in momentum space. For short-range interactions, it has been known for several years that $R(\lambda)$ satisfies a Fredholm equation whose kernel is in the Schmidt-class. The corresponding spin-orbit kernel is not in the Schmidt-class, but it is shown that it does belong to a certain class of compact operators which is larger than the Schmidt-class. A modified Fredholm theory is presented which applies to all operators in this larger class. This enables $R(\lambda)$ to be found for all values of λ in the complex plane cut along the continuous spectrum of the Hamiltonian. It is shown that the modified Fredholm theory also holds for the Coulomb interaction.

1. Introduction

In recent years our understanding of the n-body problem has considerably improved. There are now rigorous mathematical methods available to discuss non-relativistic systems consisting of any finite number of particles with two-body interactions. For three particles, a powerful approach is due to Faddeev [1, 2]. Under the assumption that the two-body scattering amplitudes are known, this gives a set of coupled equations for the three-body amplitude. Faddeev's work became widely known through two papers by Lovelace [3, 4] and was subsequently generalized to larger numbers of particles by several authors [5–9]. Alternative equations in terms of the two-body scattering amplitude were given by Rosenberg [10] and Newton [11] and applied by Noble [12]. For further information see refs. [13–15].

A different method was proposed by Weinberg [16] and further discussed by Hunziker [17]. This is closely related to a formalism which was developed independently by one of us [18]. In this formalism, it is assumed that the two-body interactions are known, and a sequence of equations is constructed which must be solved successively for the resolvents referring to $2, 3, \ldots, n$ particles. In the present paper, these