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An Elementary Proof of Dyson's Power Counting Theorem*

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Abstract. For the case of Euclidean metric an elementary proof of the power counting theorem is given.

1. Introduction

As is well known, Dyson's power counting theorem plays an important part in the theory of renormalization [1]. For the case of Euclidean metric a rigorous proof of this theorem was obtained by WEINBERG as a by-product of his work on the high-energy behavior of Feynman integrals [2]. The purpose of the present paper is to give a short, direct proof of the power counting theorem which uses more elementary methods. We restrict ourselves to the Euclidean case. An extension to the case of Minkowski metric will be discussed in a forthcoming paper.

We will be concerned with integrals of the form

$$I(q\,\mu) = \int dk \frac{P(kq)}{\prod\limits_{j=1}^{n} (l_j^2 + \mu_j^2)} \tag{1.1}$$

where

$$\begin{aligned} &q = (q_1, \ldots, q_n) && k = (k_1, \ldots, k_n) & \\ &dk = dk_1 \ldots dk_n && \mu = (\mu_1, \ldots, \mu_n) && \mu_i \geqq 0 \end{aligned} \tag{1.2}$$

with q_i, k_i denoting Euclidean four vectors. P denotes a polynomial in the components of k_i and q_i, of degree g with respect to the k_i. The four vectors l_j are of the form

$$\begin{aligned} &l = K + q\,, && K = Ck = K(k)\,, & \\ &l = (l_1, \ldots, l_n)\,, && K = (K_1, \ldots, K_n)\,, && K_j \not\equiv 0\,. \end{aligned} \tag{1.3}$$

C denotes an $n \times m$ matrix.

For integrals of the form (1.1) the following version of the power counting theorem will be proved in Section 3.

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