Commun. math. Phys. 10, 325—329 (1968)

Direction-Dependent Singularities for A^4-Coupling*

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Received June 11, 1968

Abstract. For the model of A^4-coupling it is shown that in perturbation theory a direction-dependent term is required for formulating the local field equation in limit form.

1. Introduction

In a recent paper [1] (henceforth quoted as I) a finite form of the local field equation was proposed for the model of A^4-coupling and studied in renormalized perturbation theory. The purpose of this note is to discuss certain direction-dependent singularities of the propagator which were not taken into account in I. It will be shown that these singularities lead to an additional term in the field equation of the field operator as was conjectured by K. WILSON on the basis of dimensional arguments [2]. The modified form of the field equation is[1]

$$-(\Box + m^2)\, A(x) = \lambda \lim_{\xi \to 0} j(x\,\xi) \tag{1}$$

with

$$j(x\,\xi) = \frac{:A(x+\xi)\, A(x)\, A(x-\xi): + \sigma^{\mu\nu}(\xi)\, \partial_\mu \partial_\nu A(x) - \alpha(\xi)\, A(x)}{g(\xi)}$$

$$\sigma^{\mu\nu}(\xi) = \frac{\xi^\mu \xi^\nu}{\xi^2}\, \sigma(\xi^2)$$

and

$$:A(x_1)\, A(x_2)\, A(x_3): = A(x_1)\, A(x_2)\, A(x_3) - \langle A(x_1)\, A(x_2)\rangle_0\, A(x_3) - \text{cycl. perm.}$$

for spacelike distances $(x_i - x_j)^2 < 0\; (i \neq j)$.

Unless otherwise noted the notation of I will be used throughout the present paper.

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* The research reported in this paper was supported in part by the National Science Foundation.

[1] Throughout this paper $\lim_{\xi \to 0}$ will denote the spacelike limit with $\xi^2 < 0$ and $\xi^\mu / \sqrt{-\xi^2}$ bounded.