Commun. math. Phys. 10, 280—310 (1968)

Hamilton-Jacobi and Schrodinger Separable Solutions of Einstein's Equations

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Received April 17, 1968

Abstract. This paper contains an investigation of spaces with a two parameter Abelian isometry group in which the Hamilton-Jacobi equation for the geodesics is soluble by separation of variables in such a way that a certain natural canonical orthonormal tetrad is determined. The spaces satisfying the stronger condition that the corresponding Schrodinger equation is separable are isolated in a canonical form for which Einstein's vacuum equations and the source-free Einstein-Maxwell equations (with or without a Λ term) can be solved explicitly. A fairly extensive family of new solutions is obtained including the previously known solutions of de Sitter, Kasner, Taub-NUT, and Kerr as special cases.

1. Introduction

ROBERTSON [1] and EISENHART [2] have discussed conditions under which the Hamilton-Jacobi equation and the corresponding Schrodinger equation are soluble by separation of variables in spaces which admit a complete set of mutually orthogonal families of hypersurfaces. This paper also contains a study of spaces with separable Hamilton-Jacobi and Schrodinger equations but under different conditions: firstly, the investigation is restricted to ordinary four-dimensional space-time, while being extended to include the case where the Hamilton-Jacobi and Schrodinger equations under consideration apply not only to the motions of free particles (geodesics) but also to the motions of charged particles in a Maxwell field; secondly there is the more important difference that instead of requiring that the families of hypersurfaces with respect to which separation takes place be all mutually orthogonal, we require that there be an Abelian isometry group, and that the separation should take place in such a way as to define a certain canonical tetrad of orthonormal (and not necessarily integrable) forms.

One of the purposes of requiring these separability properties for a space is to obtain sufficiently strong restrictions on it to make detailed study possible, but without imposing a high isometry group. Most of the spaces which have been studied in detail in the past have had at least three parameter isometry groups. The type of separability to be discussed here applies to spaces with (locally) two parameter Abelian iso-