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A Rigorous Formulation of LSZ Field Theory

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Abstract. An exact formulation of LSZ field theory is given. It is based on the Wightman axioms, asymptotic completeness, and a technical assumption stating the existence of retarded products of field operators. Reduction formulae are derived directly from the strong asymptotic condition. The GLZ-theorem, which states the conditions under which a given set of retarded functions defines a field theory, is formulated and proved in a rigorous way.

1. Introduction

The LSZ formulation of relativistic quantum field theory [1–3] has proved to be well adapted to the needs of elementary particle physics. In comparison with the Wightman formulation [4, 5] it suffers, however, from a lower degree of mathematical rigour. Existence problems are not always treated with due care, and limits are sometimes exchanged without full justification. Even though this lack of rigour has, up to now, not been prejudicial to the applications, it would obviously be desirable to have a rigorous version, preferably based on the Wightman axioms. Important steps in this direction have already been made. HAAG [6] and RUELLE [7] proved, starting from Wightman's axioms, the so-called strong asymptotic condition. This condition provides us with an asymptotic particle interpretation of the theory and makes it possible to add the axiom of asymptotic completeness to the familiar axioms of field theory. An exact definition of the S-matrix can then be given. HEPP [8] succeeded in deriving the LSZ reduction formulae rigorously for S-matrix elements between states with non-overlapping wave functions. These non-overlapping states form a total set in the Hilbert space of states. Nevertheless, Hepp's result is not completely satisfactory, because overlapping states are physically of considerable interest. They occur, for instance, always when in the course of a calculation the well-known summation over intermediate states has to be performed.

In this paper we propose to make some further steps towards a rigorization of LSZ. Besides the Wightman axioms and asymptotic completeness we shall have to assume the existence of retarded operators (see Chapter 3 for an exact formulation of this new postulate). On this basis we shall derive reduction formulae for the matrix elements of the field operator and the S-matrix, in the latter case also for overlapping