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Statistical Mechanics of Quantum Spin Systems. III

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Abstract. In the algebraic formulation the thermodynamic pressure, or free energy, of a spin system is a convex continuous function P defined on a Banach space $\mathfrak{B}$ of translationally invariant interactions. We prove that each tangent functional to the graph of P defines a set of translationally invariant thermodynamic expectation values. More precisely each tangent functional defines a translationally invariant state over a suitably chosen algebra $\mathfrak{A}$ of observables, i. e., an equilibrium state. Properties of the set of equilibrium states are analysed and it is shown that they form a dense set in the set of all invariant states over $\mathfrak{A}$. With suitable restrictions on the interactions, each equilibrium state is invariant under time-translations and satisfies the Kubo-Martin-Schwinger boundary condition. Finally we demonstrate that the mean entropy is invariant under time-translations.

1. Introduction

The purpose of this paper is to continue the general analysis of quantum spin systems which was presented in [1, 2] and [3]. In [2] we gave an algebraic formulation of the mathematical framework of quantum spin systems and showed that the thermodynamic pressure, or free energy, P could be considered as a convex continuous function defined on a Banach space of translationally invariant interactions. Further it was shown that the pressure also served as a generating functional of equilibrium states in the sense that the functional derivatives, i.e., the tangent functionals to the graph of P, determined translationally invariant states over a suitably chosen C^* algebra $\mathfrak{A}$ of observables. The states introduced in this manner play the same role as the more conventionally used correlation functions or thermodynamic expectation values. The results of [2] were, however, incomplete in the sense that we could only rigorously establish that P generated equilibrium states under certain restrictive conditions. In particular it was shown that if the interaction Φ were such that the tangent functional to the graph of P at Φ was unique then this tangent functional determined an equilibrium state. It was further shown that the equilibrium states obtained under such conditions described pure thermodynamic phases. This latter result