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Conformal Tensor Discontinuities in General Relativity *

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Abstract. The postulate is made that across a given hypersurface N the metric and its first derivatives are continuous. This postulate is used to derive conditions which must be satisfied by discontinuities in the Riemann tensor across N. These conditions imply that the conformal tensor jump is uniquely determined by the stress-energy tensor discontinuity if N is non-null (and to within an additive term of type Null if N is lightlike). Alternatively, $[C^{\alpha\beta}{}_{\gamma\delta}]$ and $[R]$ determine

$$\left[R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu}\right]$$

if N is non-null. These relationships between the conformal tensor and stress-energy tensor jumps are given explicitly in terms of a three-dimensional complex representation of the antisymmetric tensors. Application of these results to perfect-fluid discontinuities is made: $[C^{\alpha\beta}{}_{\gamma\delta}]$ is of type D across a fluid-vacuum boundary and across an internal, non-null shock front. $[C^{\alpha\beta}{}_{\gamma\delta}]$ is of type I (non-degenerate) in general across fluid interfaces across which no matter flows, except for special cases.

I. Introduction

The satisfaction of conditions on the stress-energy tensor alone is necessary to ensure that a discontinuity across a hypersurface be acceptable [1], and much work has been done to study such conditions [2]. The rest of the Riemann tensor – the Conformal, or Weyl tensor – therefore must have its discontinuities determined by the jump in $T^{\mu\nu}$. This fact has been known for interior-exterior junctions, and has appeared in the works of COCKE [3] and of ESTABROOK and WAHLQUIST [4]. Here we will make explicit the algebraic form of the jump in $C^{\alpha\beta}{}_{\gamma\delta}$ as given by a discontinuity in $T^{\mu\nu}$.

In Section II we will discuss the relationship between second-derivative metric jumps and Riemann tensor discontinuities. Section III will develop the language of three-dimensional complex vectors which will be used in the discussion of jumps in the Riemann tensor in Section

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