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Quasi-Free States of the C.C.R. — Algebra and Bogoliubov Transformations*

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Abstract. We give a complete characterization of quasi-free states (generalized free states) of the C.C.R. algebra. We prove that the pure quasi-free states areall Fock states and that any two Fock states are related through a symplectic automorphism (Bogoliubov transformation). We make an explicit construction of these representations which correspond to primary quasi-free states.

I. Introduction

In this work we study the set of quasi-free states on the C.C.R. algebra. The notion of quasi-free states is introduced by D. W. ROBINSON [1] in his study of the ground state of the Bose gas. Until now, one was not able to construct exactly solvable physical models, whose solutions do not belong to the set of quasi-free states. It is interesting to study this set of states in order to derive its most general properties hoping that their general properties may throw some light on the problem of construction of non-trivial models.

From a technical point of view, we start with a symplectic space (H, σ) and consider the C.C.R. C^*-algebra $\overline{\Delta(H, \sigma)}$ [2] built on it. We prove that the pure quasi-free states are all Fock states and that any pure quasi-free state can be obtained from another pure quasi-free state by acting on it through an automorphism of the algebra induced by a symplectic operator on (H, σ). The converse statement is well known by physicists as Bogoliubov transformations. Explicit representations induced by quasi-free states of C.C.R. are given. Amongst all representations we characterize the primary ones. The last property turned to be important to characterize physical systems in statistical mechanics [3]. This property was outlined by ARAKI and WOODS [4] for the temperature states of the free Bose gas which are quasi-free states.

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