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First Order Deformations of Lie Algebra Representations, E (3) and Poincaré Examples

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Abstract. A classification of "first order" deformations of Lie algebra representations by the use of a cohomology group is studied. A method is proposed for calculating this group for the case of algebras which are semi-direct products. The role of unitarity of the representations is exhibited. Applications are made for the Poincaré and $E(3)$ algebras.

Up to now, only the "first order" deformations of Lie algebra representations (connected or not with a deformation of the Lie algebra itself), seem to allow some possibilities of classification.

We recall in part I, how this is achieved by the introduction of a cohomology group $H^1(\mathcal{G}, L(V))$ where V is the (possibly infinite-dimensional) representation space. When $\mathcal{G}$ is a semi-direct product $K.T$ of a semi-simple and compact algebra K by an abelian ideal T, a general method can be used to determine this group H^1.

The procedure is exposed in part II; it is nearly the same as that which may be used for the computation of the finite dimensional representations of such algebras [1]. The application to the motion algebra $E(3)$ is straightforward, if one considers only the deformations leaving the rotation subalgebra and it representation fixed. For the Poincaré algebra we shall see, using the "Lorentz basis" that the same method can be applied (even with a non compact K).

In all the cases, we do not claim that the method used here is completely rigourous for the infinite dimensional representations — since topological questions should be discussed in that case — nevertheless we think it has at least an heuristic value.

Our main result is that the dimension of various interesting cohomology groups $H^1(\mathcal{G}, L(V))$, restricted in order to produce unitary deformations, is one on R. This is true for $SL(2, R)$, V being a representation space for the continuous series, for $E(3)$ and the Poincaré algebras, with the representations $[m, s]$, $m > 0$. It results for instance in the Poincaré case that a deformation of such a representation $[m, s]$, $m > 0$ with a fixed algebra can always change the mass, the spin being "rigid".