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Attempt of an Axiomatic Foundation of Quantum Mechanics and More General Theories. IV*

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Abstract. This contribution continues the series of papers on the same subject which has been treated by LUDWIG in [1—3]. Using the system of axioms as given in [3], we shall succeed in constructing an orthomodular lattice of linear operators on the real vector space generated by the physical decision effects. There results an isomorphism between the orthomodular lattice of all physical decision effects and the lattice to be constructed.

I. Preliminaries

As shown by FOULIS ([7—11]), any orthomodular lattice can be coordinatized by a Baer-*-semigroup (i.e. a *-semigroup where the annihilator of each element is a principal left (right) ideal generated by a self-adjoint idempotent). At this point the theory developed becomes relevant for physics: On the one hand, POOL [17, 18][1] has given the concept of a Baer-*-semigroup a direct physical meaning by including the ideal measuring process in an axiomatic lattice approach to quantum theories. On the other hand, the multiplicative semigroup in the ring of all bounded linear operators on Hilbert-space is such a semigroup. But looking for structures for physics like Hilbert-space, we notice that the mathematical situation is still more complicated: As it has generally turned out by LUDWIG'S system of axioms, the lattice to be co-ordinatized is embedded in a topological vector space. MILES [12] has given the problem the most general form: Let $\mathscr{A}$ be a B^*-algebra. Its self-adjoint elements form a real partially ordered vector space $H(\mathscr{A})$ with a positive cone of elements x^*x for all $x \in \mathscr{A}$. Required is the knowledge in how far $\mathscr{A}$ is already determined by $H(\mathscr{A})$. Ideally we should have to find the class of all B^*-algebras $\mathscr{A}$ for which for a given real partially

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