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The Classical Mechanics of One-Dimensional Systems of Infinitely Many Particles

I. An Existence Theorem

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Abstract. We prove a global existence and uniqueness theorem for solutions of the classical equations of motion for a one-dimensional system of infinitely many particles interacting by finite-range two-body forces which satisfy a Lipschitz condition.

§ 1. Introduction

In this paper, we prove an existence and uniqueness theorem for solutions of the equations of motion of a system of infinitely many classical point particles, constrained to move in one dimension, interacting by two-body forces of finite range. Thus, let (q_i, p_i) be a sequence of pairs of real numbers representing the positions and velocities of an infinite set of particles. We assume that each bounded interval in $\boldsymbol{R}$ contains only finitely many particles, and we want to solve the differential equations:

$$\frac{dq_i(t)}{dt} = p_i(t) \qquad \frac{dp_i(t)}{dt} = \sum_{j \neq i} F(q_i(t) - q_j(t)) \tag{1.1}$$

with the initial conditions:

$$q_i(0) = q_i \,; \qquad p_i(0) = p_i \,.$$

(For simplicity, we are taking the particles to be identical and to have mass one). The interparticle force F will be assumed to be bounded and to have compact support. As long as each bounded interval in $\boldsymbol{R}$ contains only finitely many $q_j(t)$'s, the sum on the right of the second equation has only finitely many non-zero terms for each i and the equations therefore make sense. It is clear, however, that for some initial configurations we must expect the Eq. (1.1) to lead in finite time to a catastrophic situation with infinitely many particles in some bounded interval. To take a trivial example, if there are no interparticle forces and if $p_i = - q_i$ for each i, then all the particles are at the origin at time one. The crux of the problem of proving an existence theorem is to find a set of initial configurations for which such catastrophies can be shown not to happen.