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A Generalization of a Theorem by Wightman

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Abstract. It is shown for some class of sets in the Minkowski space that the intersection of local Haag algebras assigned to open neighbourhoods of such a set contains only multiples of the identity.

In the paper [1] A. S. WIGHTMAN has shown for one point subset K of the Minkowski space, that the intersection of local Haag algebras [2] assigned to open neighbourhoods of the set K contains only multiples of the identity. In the present paper, this theorem is proved for a larger class of sets K. The similar results are contained in the paper [3].

For any open subset $\mathcal{O}$ in the Minkowski space X, $R(\mathcal{O})$ will mean the v. Neumann algebra of operators in the Hilbert space H, associated with $\mathcal{O}$. Let $U(\cdot)$ be the representation of the translation group. $R(\cdot)$ and $U(\cdot)$ have the following properties:

1. Translation invariance:

$$U(a)\, R(\mathcal{O})\, U(a)^{-1} = R(\mathcal{O} + a)\,.$$

2. Locality: If $\mathcal{O}$ is spacelike to $\mathcal{O}_1$, then

$$R(\mathcal{O}) \subset [R(\mathcal{O}_1)]'\,.$$

3. Spectral condition:

$$U(a) = \int e^{-iap}\, dE(p)$$

where $dE(\cdot)$ is a spectral measure with the support contained in the set $\{p: p^2 \geqq 0,\ p_0 \geqq 0\}$.

4. Uniqueness of the vacuum state: There is one and only one vector $\Omega \in H$ such that, for all translations $a \in X$: $U(a)\,\Omega = \Omega$.

5. Cyclicity of the vacuum state: The set

$$\{A\Omega : A \in R(\mathcal{O})\,,\ \mathcal{O} \subset X\}$$

is dense in H.

We shall say that a compact set $K \subset X$ fulfills the condition C, if there exist vectors $w_1, w_2 \in X$ such that: $w_1^2 = w_2^2 = -1$, $(w_1 - w_2)^2 \geqq 0$ and for all $a, b \in K$: $(a-b)^2 \leqq 0$, $|(a-b)\,w_i| \leqq \sqrt{-(a-b)^2}$ $\quad i = 1, 2$.

Theorem I. *If the compact set K fulfills the condition C, then*

$$\bigcap_{\mathcal{O} \supset K} R(\mathcal{O}) = \{\lambda I : \lambda \in C^1\}$$

where $\mathcal{O}$ runs over all open neighbourhoods of K.