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Remarks on the Quantum Field Theory in Lattice Space. II

HIROSHI EZAWA
Department of Physics, Gakushuin University, Mejiro, Toshima-ku, Tokyo

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Abstract. We consider a strong-coupling approach to $\lambda\phi^4$-meson theory as formulated in a lattice space which is of simple cubic type having lattice constant a and total volume V. Self-adjointness and regularity of the Hamiltonian are established. The strong-coupling perturbation series are examined for the cases with and without mass renormalization. The series for ground state and one-particle state as well as for their energies are shown to converge when the coupling constant is sufficiently large, say $\lambda > \lambda_c$ (sufficient condition). The bounds λ_c we have found increase with the total volume V and/or the cut-off momentum a^{-1}. Some other features of the strong-coupling perturbation theory are also discussed.

I. Introduction

This paper is to give an account of our attempt to study the perturbation theory as applied to a quantum field theory in lattice space.

The field is defined at each site of a simple cubic lattice, and it may be regarded as an average of local field over the corresponding unit cell.[1] While in the previous paper [2] we studied cases of uncoupled lattice sites, we now wish to try the strong-coupling perturbation theory [3, 4] to take care of the coupling which in fact is caused by gradient terms necessarily present in relativistic Hamiltonians. Taking up the (lattice space version of) neutral scalar field ϕ with $\lambda\phi^4$ ($\lambda \gg 1$) interaction [4], we shall use KATO's theorems (1) to establish self-adjointness of the perturbed Hamiltonian and (2) to examine convergence of the perturbation series [5, 6].

It is indeed remarkable that, if one agrees to keep the cut-off momentum as well as the volume of the "world" finite, then in the *strong*-coupling scheme KATO's theory is applicable to the $\lambda\phi^4$ theory, for in the weak-coupling scheme magnitudes of unperturbed and perturbation Hamiltonians cannot be compared in KATO's sense.

As for the latter scheme one might recall that A. M. JAFFE studied the model in *two*-dimensional space-time to prove that the weak-coupling perturbation series for Green functions are all divergent [7]; each term in the series being finite, though, in the world with such a low dimen-

[1] It may be recalled that YUKAWA proposed a hypothesis that the space consists of "elementary domains" which are not divisible any further [1].