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Attempt of an Axiomatic Foundation of Quantum Mechanics and More General Theories. III

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Abstract. Starting from axioms as physical as possible [1, 2, 3] about "effects" and "ensembles", we shall investigate further consequences.

Concerning part I and II [4, 5] the axioms can be so formulated as to be surveyed more easily.

Besides, it is possible to prove some important theorems more simply.

New structures of the lattice of decision effects are pointed out, leading in two subsequent papers at last to the final aim, the structure of Hilbert-space.

I. Summary of Former Results

After the publication of part II of this exposition it has turned out that the axioms for a foundation can be formulated still somewhat more skilfully (at least in the case of a finite-dimensional ensemble-space, see below). They can be so extended that, in fact, (finite-dimensional) Hilbert-spaces over the fields of the real or complex numbers or of the quaternions remain as irreducible solutions of the system of axioms. Therefore the axioms shall here be briefly made up once more. A physically heuristic argument can be found in [1, 2, 3]. Physical arguments more detailed will be given in the second edition of the book „Die Grundlagen der Quantenmechanik".

The starting point of our foundation are the sets $\underline{K}$ of the ensembles $\underline{V}$ and $\underline{L}$ of the effects $\underline{F}$ and a probability function μ on $\underline{K} \times \underline{L}$ satisfying:

Axiom 1.

α) $0 \leqq \mu(\underline{V}, \underline{F}) \leqq 1$,

β) $\mu(\underline{V}_1, \underline{F}) = \mu(\underline{V}_2, \underline{F})$ *for all* $\underline{F} \in \underline{L}$ *implies* $\underline{V}_1 = \underline{V}_2$,

γ) $\mu(\underline{V}, \underline{F}_1) = \mu(\underline{V}, \underline{F}_2)$ *for all* $\underline{V} \in \underline{K}$ *implies* $\underline{F}_1 = \underline{F}_2$,

δ) *there exists* $\underline{F}_0 \in \underline{L}$ *(denoted by* 0*) with* $\mu(\underline{V}, 0) = 0$ *for all* $\underline{V} \in \underline{K}$,

ε) *for each* $\underline{V} \in \underline{K}$ *there exists* $\underline{F} \in \underline{L}$ *with* $\mu(\underline{V}, \underline{F}) = 1$.

By the set of all functions on $\underline{L}$ $X(\underline{F}) := \sum_{i=1}^{n} a_i \cdot \mu(\underline{V}_i, \underline{F})$ with $\underline{V}_i \in \underline{K}$, a_i real numbers and n any finite integer, a real linear space B is defined. We will only pursue the case where B is finite-dimensional. By $X(\underline{F}) = \mu(\underline{V}, \underline{F})$, $\underline{K}$ can be indentified with a subset of B. By $\|X\| := \sup\{|X(\underline{F})| \mid \underline{F} \in \underline{L}\}$ for all $X \in B$, B becomes a normed linear space.