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The Interpretation of Some Spheroidal Metrics

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Abstract. ZIPOY recently studied some static gravitational fields in spheroidal coordinates, and endowed them with unfamiliar topologies. We examine three of these fields and show that they can also be interpreted as fields in space of Euclidean topology.

§ 1. Introduction

D. M. ZIPOY [7] has presented some static, axially symmetric solutions in spheroidal coordinates of EINSTEIN's equations for empty space

$$R_{ik} = 0 \,. \tag{1.1}$$

He endowed them with rather terrifying topological properties.

In this paper we examine three of these solutions and show that they can in fact be interpreted quite naturally as gravitational fields in space of Euclidean topology.

§ 2. The Weyl Solutions

All solutions studied are members of the Weyl static axially symmetric class [4]. The line element in pseudo cylindrical polar coordinates is

$$ds^2 = -e^{2(\lambda-\sigma)}(dz^2 + d\varrho^2) - \varrho^2 e^{-2\sigma} d\phi^2 + e^{2\sigma} dt^2 \,, \tag{2.1}$$

$$(-\infty < z < \infty,\ 0 \leqq \varrho < \infty,\ 0 \leqq \phi \leqq 2\pi,\ -\infty < t < \infty)\,,$$

where λ and σ are functions of z and ϱ. Following ZIPOY, we transform to pseudo oblate spheroidal coordinates by putting

$$z = a \sinh u \sin\theta, \quad \varrho = a \cosh u \cos\theta, \quad \phi = \phi, \quad t = t\,, \tag{2.2}$$

a being a constant; (2.1) then becomes

$$\begin{aligned} ds^2 = &-a^2 e^{2(\lambda-\sigma)}(\sinh^2 u + \sin^2\theta)(du^2 + d\theta^2) \\ &- a^2 e^{-2\sigma} \cosh^2 u \cos^2\theta \, d\phi^2 + e^{2\sigma} dt^2 \,. \end{aligned} \tag{2.3}$$

ZIPOY takes the ranges of the coordinates as follows

$$0 \leqq u < \infty\,, \quad -\frac{\pi}{2} \leqq \theta \leqq \frac{\pi}{2}, \quad 0 \leqq \phi \leqq 2\pi, \quad -\infty < t < \infty\,. \tag{2.4}$$

As is well known, if (2.1) is a solution of (1.1) σ has to satisfy LAPLACE's equation; for example in the coordinates of (2.3) σ must satisfy

$$\frac{\partial^2\sigma}{\partial u^2} + \frac{\partial^2\sigma}{\partial\theta^2} + \tanh u \frac{\partial\sigma}{\partial u} - \tan\theta \frac{\partial\sigma}{\partial\theta} = 0\,. \tag{2.5}$$