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Remarks on the Quantum Field Theory in Lattice Space. I

HIROSHI EZAWA
II. Institut für Theoretische Physik, Universität Hamburg

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Abstract. We calculate the Gelfand functionals $E(f, g; a)$ for quantized fields ϕ in lattice space, a being the lattice constant. In the limit $a \to 0$ the functionals take on two different forms depending upon the "potential" $F[\phi]$ of the lattice Hamiltonian (coupling between different lattice sites not included). If $F[\phi]$ is of a short-range type (see text for definition) the limit functional is Gaussian. The corresponding representation of CCR is reducible and its realization apparently non-unique unless $F[\phi]$ is quadratic. The most natural realization is to represent the field as a linear combination of Fock fields whose masses are given by the excitation energies of the lattice Hamiltonian. If $F[\phi]$ is of a long-range type, the limit functional takes the more general form once studied by ARAKI.

I. Introduction and Summary

As an object of quantum mechanics the field is distinguished from any (finite) particle systems by the infinity of its degrees of freedom. It is sometimes asserted that the quantized local field should be dealt with as a limit of some *approximate* field [1] (finite box, finite cut-off or averaged bilocal interaction [2], [3]).

In the present series of papers we choose to consider the limit of a quantized field in a lattice space, by which we mean a set of canonical variables $\{\pi(\mathbf{s}), \phi(\mathbf{s})\}$ defined for each site $\mathbf{s}$ of a discrete lattice (simple cubic, lattice constant a, total volume $V < \infty$). We assume the commutation relations, (see [3]):

$$[\pi(\mathbf{r}), \phi(\mathbf{s})] = -ia^{-3}\delta_{r,s}, \qquad \text{etc.} \tag{1.1}$$

Deferring the discussion of a coupling between different lattice sites to the subsequent paper, the first paper deals with the case of no coupling, the Hamiltonian of the system being of the form,

$$\mathscr{H}_0 = a^3 \sum_s \left\{\frac{1}{2}\pi(\mathbf{s})^2 + F[\phi(\mathbf{s})]\right\}. \tag{1.2}$$

We study the limit representation at $a \to 0$ of the canonical commutation relations in the following way: (1) We calculate the ground-state expectation functional $E(f, g; a)$ for the lattice field. (2) We let a approach zero