

# Master Analytic Representations and Unified Representation Theory of Certain Orthogonal and Pseudo-Orthogonal Groups

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Received August 8, 1967

**Abstract.** The representation theory of the groups  $SO(5)$ ,  $SO(4, 1)$ ,  $SO(6)$  and  $SO(5, 1)$  is studied using the method of Master Analytic Representations (MAR). It is shown that a single analytic expression for the matrix elements of the generators of  $SO(n + 1)$  and  $SO(n, 1)$  in an  $SO(n)$  basis yields all the unitary representations (for  $n = 4, 5$ ); and that the compact and non-compact groups have essentially the same analytic representation. Once the MAR of a group is worked out, the search for the unitary irreducible representations is reduced to a purely arithmetic operation. The utmost care has been exercised to conduct the discussions at an elementary level: knowledge of simple angular momentum theory is the only prerequisite.

## Introduction

In the course of a study of the unitary irreducible representations of a variety of groups we have discovered that, in all cases where the representations have been analyzed in detail, in every linear representation of the (locally compact) Lie algebra, the matrix elements describing the representation was a specialization of certain analytic functions. We refer to this general representation as the Master Analytic Representation (MAR). It provides us with a method of studying the representations of various noncompact groups. In this paper we apply this method to study the representations of the groups  $SO(5)$ ,  $SO(4, 1)$ ,  $SO(6)$  and  $SO(5, 1)$ .

The plan of the paper is as follows: in Sec. 1 we study the MAR for  $SO(5)$  in an  $SO(4)$  basis and deduce all the unitary representations of the  $SO(5)$  group [1]. An elementary diagrammatic analysis for this purpose is developed and exploited. We then use the Weyl trick [2] to write down the MAR for the de Sitter group  $SO(4, 1)$  and specialize it to find all the unitary irreducible representations of  $SO(4, 1)$  and of its covering group [3]. Sec. 2 deals with the unitary representations of the  $SO(5, 1)$  group

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\* Work supported in part by the National Science Foundation.

\*\* Work supported in part by the U.S. Atomic Energy Commission.

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