Commun. math. Phys. 8, 185—203 (1968)

A Generalized Edge of the Wedge Theorem

Å. KOLM and B. NAGEL
Department of Theoretical Physics, Royal Institute of Technology, Stockholm

Received September 1, 1967

Abstract. The edge of the wedge theorem is generalized to the case where one only assumes the existence and equality of the distribution boundary values of $f_\pm(z)$ and all their derivatives on some analytic curve $\mathscr{C}$ in R^n. Here $f_\pm(z)$ are holomorphic in $R^n \pm iC$, respectively, where C is a convex cone, and $\mathscr{C}$ has its tangent vector in C at every point. Under these assumptions there exists an analytic continuation $f(z)$ holomorphic in some complex neighbourhood of the double cone generated by $\mathscr{C}$. A proof is also given of the connection between the existence of a distribution boundary value and the growth of the holomorphic function near the boundary.

I. Introduction

The edge of the wedge theorem is an often used tool in quantum field theory (see [1], Ch. 2–5, and [2], § 27, for proof, discussion, and references). This theorem states that given two functions $f_\pm(z)$ of n complex variables $z = (z_1, \ldots, z_n)$, holomorphic in the two tubes $R^n \pm iC$, respectively (C is an open convex cone with vertex in the origin), and having equal boundary values in the distribution sense on some open set Ω in R^n, there is a common analytic continuation $f(z)$ of $f_\pm(z)$, holomorphic in some complex neighbourhood of Ω. (We are not concerned here with the case studied by EPSTEIN [3], where one has two cones C_1 and C_2 with $C_1 \cap (-C_2) = \emptyset$.) As derivation is a continuous operation on distributions, existence and equality of the boundary values of $f_\pm(z)$ on Ω imply the existence and equality of the boundary values of all pairs of derivatives $f_\pm^{(p)}(z)$ on Ω.

An extension of the edge of the wedge theorem in the case $n \geqq 2$ is the theorem of the C-convex hull ([2], § 28; [4]), stating that under the assumptions above the holomorphy domain of $f(z)$ can be enlarged to contain a complex neighbourhood of the C-convex hull $B_C(\Omega)$ of Ω. Here $B_C(\Omega)$ is the smallest set containing Ω, such that if the segment $[x, x']$ belongs to $B_C(\Omega)$ then so does the double cone $(x + C) \cap (x' - C)$. This result is proved with analytic completion techniques (continuity theorem).

It might be of interest to ask what conclusions can be drawn if we assume only the equality of the boundary values of $f_\pm(z)$ and all derivatives on some lower-dimensional subset of Ω. The first difficulty is what