Commun. math. Phys. 8, 164—184 (1968)

Number Operators for Representations of the Canonical Commutation Relations

Jan M. Chaiken
Cornell University, Ithaca, N. Y.*
Massachusetts Institute of Technology, Cambridge, Mass.**

Received December 30, 1967

Abstract. A *number operator* for a representation of the canonical commutation relations is defined as a self-adjoint operator satisfying an exponentiated form of the equation $Na^* = a^*(N + I)$, where a^* is an arbitrary creation operator. When N exists it may be chosen to have spectrum $\{0, 1, 2, \ldots\}$ (in a direct sum of Fock representations) or $\{0, \pm 1, \pm 2, \ldots\}$ (otherwise). Examples are given of representations having number operators, and a necessary and sufficient condition is given for a direct-product representation to have a number operator.

Introduction

The Fock representation of the canonical commutation relations has a total occupation number operator N. One way of completely describing N is to say

(i) it is self-adjoint

(ii) its spectrum is $\{0, 1, 2, \ldots\}$

and

(iii) it satisfies the commutation relation,

$$N a^*(\varphi) = a^*(\varphi)\,(N + I) \tag{0.1}$$

in a suitably rigorous form. Here $a^*(\varphi)$ is the creation operator for a wavefunction φ, and (0.1) is to hold for all φ.

In fact, the only representations of the canonical commutation relations which have a number operator N satisfying (i)—(iii) are direct sums of Fock representations [2, 4, 5].

If we relax the requirements on N by eliminating the assumption (ii) about the spectrum, then there exist other representations of the canonical commutation relations possessing such number operators. We call them *particle representations*.

In Section 1 we discuss general properties of particle representations. For a *strange* particle representation (other than a direct sum of Fock

* Research supported in part by National Science Foundation Grant GP 5882.
** Address for 1967-8. Research supported by AFOSR Grant 44620-67-C-0008.