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Matrix Elements of Representations of Non-Compact Groups in a Continuous Basis*

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Abstract. Explicit formulas are obtained by a simple algebraic method for the representations of the finite group transformations of $O(2,1)$ in a continuous basis when a non-compact generator is diagonalized. Compact and non-compact cases are treated in a unified form and the nature of analytic continuation is determined. The transformation function between the discrete and the continuous bases is also given. These explicit formulas have not been obtained in the literature before.

I. Introduction

The use of the unitary infinite-dimensional representations of non-compact groups to describe the properties of bound states of quantum-mechanical systems is by now well understood. However, not much has been done to deal with the scattering states. There is a need for explicite forms of unitary representations when a continuous spectrum is used to label the states (i.e. diagonalized). There arise here some peculiar and unfamiliar (at least to physicists) problems that must be solved.

There has been a number of recent discussions on the unified representation theory of compact and non-compact groups having the same complex extention [1—3]. A number of recent papers deal with the specific cases of the representations of $O(3,1)$ with respect to the non-compact group $O(2,1)$ and the analytic continuation problem between $O(2,1)$ and $O(3)$ [1—3, 8—10]. In none of the previous work there appears the explicite form of the representation in a continuous basis and the relation of the continuous basis to the discrete one. The purpose of this work is to fill this gap. The simple algebraic method that we give in this paper as an extension of the previous work [3] not only determines the representations in a continuous basis $|\lambda\rangle$, but also gives explicite formulae for the transformation function $\langle m \mid \lambda\rangle$ between the discrete basis $|m\rangle$ and the continuous basis $|\lambda\rangle$, and the matrix elements $\langle\lambda |U| \lambda'\rangle$, where

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