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On the Asymptotic Exactness of the Bogoliubov Approximation for Many Boson Systems*

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Abstract. The Bogoliubov approximation for many boson systems consists in replacing the field operators a_0 and a_0^+ by c-numbers, to be determined by an extremum condition. Here we formulate the approximation in terms of coherent states of the condensed particles, and prove that for reasonable interactions it gives the exact values of the thermodynamical functions in the infinite volume limit.

Introduction

The standard procedure to describe a many boson system in thermal equilibrium, exhibiting Bose Einstein condensation, is the Bogoliubov approximation (hereafter called B.A.), which was proposed as early as 1947, [1]. One takes advantage of the macroscopic occupation of the zero momentum one particle state to replace the corresponding creation and annihilation operators a_0 and a_0^+ by c-numbers. The replacement is supposed to be exact in the infinite volume limit. The validity of the B.A., however, has not been rigorously proved, and has even been questioned [2]. Arguments to support it have been given in the case of the ground state energy, with special types of interactions [3, 4]. In the present paper, we want to make a first step towards a general proof of its asymptotic exactness in the limit of infinite systems.

In order to show how the present work fits into the general scheme, we first review the latter briefly and qualitatively, following BOGOLIUBOV [5] and HAAG [6]. We consider a many boson system in equilibrium, at a given non-zero temperature T ($\beta = 1/kT$), a given chemical potential μ, and with given interactions. The system is described in the grand canonical formalism (that this is at all possible will be shown in Sec. 1). We are interested in the infinite volume limit, with possible occurrence of Bose-Einstein condensation.

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