Commun. math. Phys. 8, 12—25 (1968)

Boson Fields with Nonlinear Selfinteraction in Two Dimensions*

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Received December 15, 1967

Abstract. Semiboundedness of the total Hamiltonian is proved for a selfinteracting Boson field in two dimensional space time. The interaction is given by a Wick polynomial: $P(\Phi)$:. The polynomial P is required to have even degree and its leading coefficient must be positive. A space cutoff is introduced in the interaction Hamiltonian.

§ 1. Introduction

In [10] NELSON considered the following problem. Let Φ be a neutral scalarfield of mass $\mu_0 > 0$ in two dimensional space time and let

$$H = H_0 + g \int : \Phi^4(x) : dx \,, \tag{1.1}$$

where H_0 is the free Hamiltonian for the mass μ_0 and $g > 0$. If the system is placed in a box with periodic boundary conditions then Nelson proved that H is bounded from below. H thus has a natural selfadjoint semibounded extension (the Friedrichs extension), which can (presumably) be used to solve the Schrödinger equation. In [5], Jaffe considered the related Hamiltonian

$$H = H_0 + \int : P(\Phi(x)) : h(x)\, dx \,, \tag{1.2}$$

P a polynomial, again in two dimensional space time. Jaffe showed H to be a symmetric densely defined operator; no box is needed here. In this paper we apply Nelson's method to Jaffe's Hamiltonian (1.2). Our main result is

Theorem A. *Let h be a nonnegative function in $L_1 \cap L_2$. Suppose that the polynomial P in* (1.2) *has even degree and that the leading coefficient is positive. Then H is bounded from below.*

By elementary methods we also show that the Hamiltonian

$$\varepsilon N + \int : \Phi^2(x) : h^2(x)\, dx \tag{1.3}$$

is bounded from below, where ε is any positive number and N is the number of particles operator. This bound on (1.3) permits an improve-

* This work was supported in part by the National Science Foundation, NSF GP 7477