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A Note on the Decrease of Truncated Wightman Functions for Large Space-like Separation of the Arguments

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Abstract. The truncated Wightman functions cannot decrease arbitrarily fast for large space-like separation of the arguments. For certain configurations they can fall off at most exponentially.

Upper bounds on the decrease of truncated Wightman functions were established a long time ago [1—5]. For instance, for a relativistic quantum field theory of a self-interacting neutral, scalar field $A(x)$ H. ARAKI [2] (compare the footnote in [5]) proved the following theorem: Under the assumptions of a) Lorentz invariance, b) temperedness of the Wightman functions, c) the existence of a lowest non-zero mass, the truncated vacuum expectation value (TVEV)

$$\langle A(x_0) \ldots A(x_n)\rangle^T$$

vanishes *at least* exponentially for $x_{i-1} - x_i = \xi_i + \lambda \xi_i'$ $i = 1, \ldots, n$ where $\xi_i + \lambda \xi_i'$ should be a Jost point for sufficiently large λ and $\lambda \to +\infty$, ξ_i, ξ_i' fixed (with at least one $\xi_i' \neq 0$).

Here we want to point out that a *lower* bound on the decrease of the TVEV for similar configurations can be obtained as well. We do not assume locality or the existence of a lowest non-zero mass.

To begin with, let us consider the 2-point function. Lorentz invariance, temperedness and positive definiteness imply the well-known Källén-Lehmann representation

$$\langle A(x_0)\, A(x_1)\rangle^T = \langle A(x_0)\, A(x_1)\rangle = i \int_0^\infty d\varrho(\mu)\, \Delta_\mu^+(x_0 - x_1)\,,$$

$\varrho(\mu)$ a positive tempered measure

$$\sim \frac{\text{const}}{-(x_0 - x_1)^2}$$

$$\left(\text{or} \sim \frac{\sqrt{m}}{2^{5/2}\,\pi^{3/2}\sqrt{-(x_0 - x_1)^2}^{3/2}} \exp\{-m\sqrt{-(x_0 - x_1)^2}\} \text{ in case of the existence of a lowest non-zero mass } m \text{ in the theory}\right).$$

Next, we turn to the 3-point function. It is analytic in the "extended tube" $\mathscr{T}'_{0,1,2}$ the boundaries of which are explicitly known in terms of the invariants [6]. Consider

$$W_2^T(x_0, x_1, x_2) = \langle A(x_0)\, A(x_1)\, A(x_2)\rangle^T$$