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Application of the Riemann Method to the Bethe-Salpeter Equation

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Abstract. The Bethe-Salpeter equation describing the interaction of two scalar particles via the exchange of a third scalar particle with mass $\mu \neq 0$ is in configuration space a hyperbolic partial differential equation of fourth order which will be studied with the help of the Riemann method. This method yields two Volterra equations the solutions of which are special solutions of the Bethe-Salpeter equation. The wave function is a superposition of the special solutions. For the coefficients one gets a system of two integral equations. The Fredholm determinant of the system is the generalization of the nonrelativistic Jost function.

1. Introduction

An exhaustive treatment of the Schrödinger equation has been given by Newton. Crucial for the success of this method is the introduction of several modified Green's functions leading to Volterra integral equations. The Volterra equations can be solved by iteration for all values of the potential-strength. Despite the fact that the Schrödinger equation is an ordinary differential equation while the Bethe-Salpeter equation is a partial differential equation the generalization of this method to the Bethe-Salpeter case is possible. The Volterra equations in two variables can be established with help of the Riemann method [2, 3] or formally by splitting the Green's function into a Riemann function and two residual terms. The solutions of the integral equations which can be obtained by iteration are special solutions of the Bethe-Salpeter equation. The solution with causal boundary conditions is a superposition of the special solutions. For the coefficients in this expansion we get a system of two integral equations in one variable. The Fredholm determinant of the system is the generalization of the nonrelativistic Jost function [4].

In Sec. II we treat the radial Schrödinger equation. For convenience we confine us as in the Bethe-Salpeter case to zero angular momentum. Only those aspects are written down which can be already generalized. In Chapt. 3 we write down the differential form of the Bethe-Salpeter equation for two scalar particles with masses m_1 and m_2 which interact via a potential. Here we have in mind the Yukawa potential which de-