Commun. math. Phys. 7, 190—217 (1968)

Spectral Representations of Lorentz Invariant Distributions and Scale Transformation*

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Received June 15, 1967

Abstract. An approach to the theory of Lorentz invariant distributions is developed in terms of covariant spectral representations. The behaviour of singular invariant distributions under a change of scale is analyzed. It is shown that the conventional extension of homogeneous singular functions into distributions in R^4, followed by a breakdown of homogeneity, is incomplete. Homogeneous extensions depending on an arbitrary scaling parameter are introduced, calculation techniques are developed and various formulae having applications in quantum field theory are derived.

1. Introduction

The aim of this paper is to present a new approach to Lorentz invariant distributions in terms of spectral representations which exhibit the covariant form of the functionals, permit to overcome the "origin of the light cone" difficulties and lead to a considerable simplification of calculation techniques. Thus, the approach is possibly simpler than that of Refs. [1]—[5]. On the basis of this formalism we investigate the behaviour of certain singular invariant distributions under a change of scale: The conventional way of associating distributions in R^4 with homogeneous singular functions by regularization gives rise to functionals which are no longer homogeneous. Such inhomogeneous distributions (e.g. the propagators $(x^2 - i0)^{-n}$, $n \geqq 2$) are physically unacceptable because the space-time or momentum variables on which they depend carry dimension. The extension of singular homogeneous functions into homogeneous distributions in R^4 requires the introduction of an arbitrary scaling parameter bearing dimension. Then, a breakdown of dilatation symmetry by regularization can be avoided if a similarity transformation in space-time is accompanied by a corresponding change of this scaling parameter.

In Sec. 2 we develop the theory of Lorentz invariant distributions in terms of spectral representations. In Sec. 3 we analyze the problem

* Supported by the Deutsche Forschungsgemeinschaft.