Commun. math. Phys. 7, 160—163 (1968)

On Subalgebras which Survive Contraction*

GABOR DOMOKOS** and GRAHAM L. TINDLE***

Department of Physics and Lawrence Radiation Laboratory
University of California, Berkeley

Received September 1, 1967

Abstract. We prove the following theorem: if a subalgebra **B** of an algebra **G** is spanned by root vectors, then if X is a regular element of **G**, the limit $\lim_{t\to\infty} \operatorname{Ad} \exp tX(\mathbf{B})$ exists and is isomorphic to **B**, i.e. **B** "survives" contraction with X. The algebra $\mathbf{SL}(2C)$ is considered as an example. In particular it is shown that $\mathbf{SL}(2C)$ itself survives and applications to relativistic scattering theory are indicated.

1. Introduction

The investigation of the contraction of Lie groups [1, 2] is of considerable interest in theoretical physics. Apart from the classical examples, (e.g. contraction of the Poincaré group to the Galilean group in the limit of infinite light velocity) it was recently realized [3, 4] that group contraction is an important concept in a relativistic scattering theory.

It is well known that the little group (the isotropy group of the four momentum P) of the Poincaré group is isomorphic to $O(3)$, or $SU(2)$ if we wish to describe half-integral spins, provided the total four momentum, P, is time-like. As the total invariant mass of a system, $\sqrt{P^2}$, tends to zero, the group $SU(2)$ contracts to $E(2)$, the group of rigid motions in a plane. The sudden change in the structure of the little group at $P^2 = 0$ gives rise to certain singularities in the partial wave amplitudes, which contradict the conventionally assumed analytic properties of the scattering amplitude. It may be shown [4] that these "unwanted" singularities can be eliminated if, at the contraction point, the partial wave amplitudes, or rather the Regge-pole terms, can be grouped together to form families which transform according to representations of a group which does not change its structure at the contraction point.

* Research supported in part by the Air Force Office of Scientific Research, Office of Aerospace Research, under Grant No. AF-AFOSR-232-66.

** On leave of absence from the Central Research Institute for Physics. Address after September 15, 1967: Central Research Institute for Physics, Budapest, Hungary.

*** N.A.T.O. post-doctoral fellow.