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Algebras of Observables with Continuous Representations of Symmetry Groups*

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Abstract. *Concrete C^*-algebras*, interpreted physically as algebras of observables, are defined for quantum mechanics and local quantum field theory.

A *quantum mechanical system* is characterized formally by a continuous unitary representation up to a factor U_g of a symmetry group $\mathfrak{G}$ in Hilbert space $\mathfrak{H}$ and a von Neumann algebra $\mathfrak{R}$ on $\mathfrak{H}$ invariant with respect to U_g. The set $\mathfrak{A}$ of all operators $X \in \mathfrak{R}$ such that $U_g X U_g^{-1}$, as a function of $g \in \mathfrak{G}$, is continuous with respect to the uniform operator topology, is a C^*-algebra called the *algebra of observables*. The algebra $\mathfrak{R}$ is shown to be the weak (or strong) closure of $\mathfrak{A}$.

In *field theory*, a unitary representation up to a factor $U(a, \Lambda)$ of the proper inhomogeneous Lorentz group $\mathfrak{G}$ and local von Neumann algebras $\mathfrak{R}_C$ for finite open space-time regions C are assumed, with the usual transformation properties of $\mathfrak{R}_C$ under $U(a, \Lambda)$. The collection of all $X \in \mathfrak{R}_C$ giving uniformly continuous functions $U(a, \Lambda) X\, U^{-1}(a, \Lambda)$ on $\mathfrak{G}$ is then a local C^*-algebra $\mathfrak{A}_C$, called the *algebra of local observables*. The algebra $\mathfrak{A}_C$ is again weakly (or strongly) dense in $\mathfrak{R}_C$. The norm-closed union $\mathfrak{A}$ of the $\mathfrak{A}_C$ for all C is called *algebra of quasilocal observables* (or quasilocal algebra).

In either case, the group $\mathfrak{G}$ is represented by automorphisms $\mathbf{V}_g$ resp. $\mathbf{V}(a, \Lambda)$ — with $\mathbf{V}_g X = U_g X U_g^{-1}$ — of the C^*-algebra $\mathfrak{A}$, and this is a *strongly continuous representation* of $\mathfrak{G}$ on the Banach space $\mathfrak{A}$. Conditions for $\mathbf{V}(a, \Lambda)$ can then be formulated which correspond to the usual *spectrum condition* for $U(a, \Lambda)$ in field theory.

1. Introduction and Summary

In quantum mechanics, physical quantities (observables) are represented by Hermitean operators A on a certain Hilbert space $\mathfrak{H}$. If moreover these observables are suitably selected, they can be represented by bounded operators A. Implicitely or explicitely, most theoretical investigations also assume the inverse: Any bounded Hermitean operator A on $\mathfrak{H}$ compatible with the superselection rules (i.e., commuting with all "superobservables") of the theory is supposed to represent a physical observable. The set of observables then coincides with the set of all Hermitean operators of a certain von Neumann algebra $\mathfrak{R}$. In field theory, the introduction of local von Neumann algebras $\mathfrak{R}_C$ for all

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