Commun. math. Phys. 7, 55—76 (1968)

States on Clifford Algebras

ERIK BALSLEV* and ANDRÉ VERBEURE**
Institut des Hautes Etudes Scientifiques
91 Bures-sur-Yvette — France

Received July 10, 1967

Abstract. We study states on Clifford algebras from the point of view of C*-algebras. A criterium is given under which the odd-point functions vanish. A particular set of states, called quasi-free states is extensively studied and explicit representations are given; as an application we give an approximate calculation of the ground state of a Fermion system.

I. Introduction

Recently quantum field theory and statistical mechanics have been studied from the point of view of C*-algebras. The key idea is the following. The set of quasi-local observables of a physical system forms a C*-algebra, and the physical states of the system correspond to the states (positive linear functionals) on the C*-algebra.

The C*-algebra formed by the field variables of a Bose field is studied by D. KASTLER [1] and D. W. ROBINSON [2].

In this paper we study states on the Clifford algebra formed by the field varables of a Fermion field.

In section II, containing the definition of a C*-Clifford algebra and several relevant notions originating from physical considerations we prove that the odd-point functions vanish for states invariant under a locally compact group, a property which was known for relativistic field theories.

In section III we give a definition of a particular class of states, called quasi-free states. This notion has been introduced by D. W. ROBINSON [2] for Bose systems. Here we define such states for Fermion fields. It is proved that the set of quasi-free states can be described by means of the set of pairs (R, S) of operators on the test function space (see theorem 2); gauge invariant quasi-free states are characterized by $S = 0$ and translation invariant quasi-free states by convolution operators R and S defined by distributions whose Fourier transforms are essentially bounded.

---

* On leave from Matematisk Institut, University of Aarhus, Denmark.

** Aangesteld navorser van het Belgisch N. F. W. O. On leave from University of Louvain, Belgium.