Commun. math. Phys. 7, 51—54 (1968)

A Geometrical Model Showing the Independence of Locality and Positivity of the Energy

S. Doplicher*
Palmer Physical Laboratory, Princeton University, N. J.

T. Regge
Institute for Advanced Study, Princeton, N. J.

and I. M. Singer**
M. I. T., Mathematics Department, Cambridge, Mass.

Received June 1, 1967

Abstract. A model of local rings system is constructed which satisfies all the usual postulates but has no representation where the energy momentum spectrum lies in the forward light cone.

1. Introduction

We consider local rings systems described by a C^*-algebra $\mathfrak{A}$, the algebra of quasilocal observables [1], a mapping $O \to \mathfrak{A}(O)$ which assigns to each bounded open region in space-time the C^*-subalgebra of $\mathfrak{A}$ of the observables localized in O, and a representation $x \to \alpha_x$ of the space-time translation group T in the *-automorphisms group of $\mathfrak{A}$ such that

$\mathfrak{A}$ is simple with unit; (1)

$\mathfrak{A}(O_1) \subseteq \mathfrak{A}(O_2)$ if $O_1 \subseteq O_2$ (isotony),
$AB = BA$ all $A \in \mathfrak{A}(O_1)$, $B \in \mathfrak{A}(O_2)$ if $O_1 - O_2$ is spacelike (locality), $\bigcup_0 \mathfrak{A}(O)$ is dense in $\mathfrak{A}$; (2)

$\alpha_x(\mathfrak{A}(O)) = \mathfrak{A}(O + x)$, $x \in T$; (3)

$x \to \alpha_x(A)$, $A \in \mathfrak{A}$, is continuous from T into $\mathfrak{A}$ [1]; (4)

if Δ is the region between any two assigned equal time hyperplanes, then (5)
$\bigcup_{0 \subseteq \Delta} \mathfrak{A}(O)$ generates $\mathfrak{A}$ (time slice axiom) [4].

We say that a representation π of $\mathfrak{A}$ on the Hilbert space $\mathfrak{H}$ satisfies the spectrum condition if there exists a unitary continuous representa-

* On leave from Istituto di Fisica G. Marconi, Università di Roma.

** Work supported by the National Science Foundation (Contract GP-6166).

[1] For a discussion and motivation of (4) see ref. [2]; for some special consequences of it see [3]. Though (4) was not explicitly stated in [5], some implications of it were used there.