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On the Primitive Characters of the Poincaré Group

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Abstract. We calculate explicitely the traces of the different types of irreducible representations of the Poincaré group. These have the form of generalized class functions and their mathematical structure follows from a generalization to non-compact groups of Frobenius' construction of induced characters.

I. Introduction

Representation theory of the Poincaré group $\mathfrak{P}$ plays a fundamental role in relativistic kinematics of elementary particles [1]. Therefore it is natural to ask which of the well-known techniques of representation theory can be developped for this group. In this paper we study the problem of the characters of the irreducible representations of $\mathfrak{P}$.

The character $\chi(g)$ of a finite dimensional representation of a finite or compact group $G = \{g\}$ by linear transformations $T(g)$ is defined as

$$\chi(g) = \operatorname{Trace} T(g) . \tag{1.1}$$

Equivalent representations have the same characters. The characters $\chi^m(g)$ of irreducible representations are called primitive. The set of primitive characters $\{\chi^m(g)\}$ is called the dual space $\hat{G}$ of G.

The properties

$$\chi(fgf^{-1}) = \chi(g) \quad f, g \in G \tag{1.2}$$

and

$$\chi(gf) = \chi(fg) \tag{1.3}$$

respectively follow immediately. Functions with property (1.2) being constant on conjugation classes are called class functions.

In the theory of representations of finite or compact groups it is shown [2], that the primitive characters are idempotent with respect to convolution

$$\int_G d\mu(f)\, \chi^m(gf^{-1})\, \chi^n(f) = \frac{\delta_{mn}\, \chi^m(g)}{\dim(m)} . \tag{1.4}$$

Here $d\mu(f)$ denotes the normalized Haar measure of G, the dimension of the representation with character $\chi^m(g)$ is $\dim(m)$. Because every representation of a compact group is equivalent to a unitary one, for