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Complete Sets of Solutions of Linear Lorentz Covariant Field Equations with an Infinite Number of Field Components

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Abstract. We study field equations of the Gelfand-Yaglom type

$$\left(- i \Gamma_\mu \frac{\partial}{\partial x_\mu} - \varkappa\right) \Psi(x) = 0$$

where Ψ transforms as a unitary representation of the inhomogeneous Lorentz group. We construct a complete set of solutions of this equation. This set includes solutions with spacelike momentum. Our method makes use of the decomposition of unitary representations of the homogeneous Lorentz group into unitary representations of the little groups $SU(2)$ and $SU(1, 1)$. The covariant operators Γ_μ are written as differential operators on homogeneous spaces. For some classes of equations we calculate the mass spectrum explicitly.

I. Introduction

We study equations of the type

$$\left(- i \Gamma_\mu \frac{\partial}{\partial x_\mu} - \varkappa\right) \Psi(x) = 0$$

where $\varkappa$ is a real parameter different from zero, and Ψ transforms as a finite direct sum of unitary irreducible representations of the homogeneous Lorentz group. This means that for any element "a" of the homogeneous Lorentz group $SL(2, C)$, we have a transformation

$$\Psi(x) \underset{a}{\longrightarrow} \Psi'(x) ,$$

$$\Psi'(x) = U_a \Psi(\Lambda_a^{-1} x)$$

such that Ψ is a vector in a representation space of $SL(2, C)$ depending on the argument x and U_a is a unitary operator in this space representing the element "a". Further we define

$$(\Lambda_a x)_\mu = \Lambda_{a\,\mu}^{\;\nu} x_\nu .$$

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