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Euclidean Proof of the Goldstone Theorem*

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Abstract. The Goldstone theorem in the formulation of KASTLER, ROBINSON, and SWIECA is proven in the framework of Euclidean quantum field theory. One utilizes that Schwinger functions have the cluster property in all directions.

There is no lack of proofs of the Goldstone theorem [1]. We present here a proof of the theorem as formulated by KASTLER, ROBINSON and SWIECA [2] in the framework of Euclidean quantum field theory [3] derived [4, 5] from a Wightman theory [6]. Although such theory requires more assumptions than are known to suffice [2] for the Goldstone theorem, the simplicity of the proof itself is noteworthy and exhibits a feature of the Euclidean approach which is interesting even for Lagrangian-free field theory.

We consider the Wightman theory of a scalar multicomponent field $A(x)$ where we suppress the subscript. Nonscalar fields can be dealt with similarly as below and require merely a straightforward extension of notation. We assume the existence of a local automorphism $A(x) \to$ $\to A^{\tau}(x)$ which is generated by a conserved current:

$$(\partial/\partial\tau)\, A^{\tau}(x) = i\,[Q_V(x^0), A^{\tau}(x)] \tag{1a}$$

$$Q_V(x^0) = \int_V d^3x'\, j^0(x^0, \mathbf{x}')\,, \quad \mathbf{x} \in V \tag{1b}$$

$$\partial_\mu j^\mu(x) = 0\,. \tag{2}$$

Examples of such automorphisms acting e.g., on the suppressed subscript are one-dimensional subgroups of constant-parameter gauge groups or of internal-symmetry groups. We assume the current operator to be local and to transform as a vector field. We choose a C^∞ time-smearing function $f(t)$ with support in $(-T, +T)$ and obeying $\int f(t)\,dt = 1$ and

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