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Types of von Neumann Algebras Associated with Extremal Invariant States

ERLING STØRMER
Mathematical Institute, University of Oslo, Oslo, Norway

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Abstract. A globalized version of the following is proved. Let $\mathscr{R}$ be a factor acting on a Hilbert space $\mathscr{H}$, G a group of unitary operators on $\mathscr{H}$ inducing automorphisms of $\mathscr{R}$, x a vector separating and cyclic for $\mathscr{R}$ which is up to a scalar multiple the unique vector invariant under the unitaries in G. Then either $\mathscr{R}$ is of type III or ω_x is a trace of $\mathscr{R}$. The theorem is then applied to study the representations due to invariant factor states of asymptotically abelian C^*-algebras, and to show that in quantum field theory certain regions in the Minkowski space give type III factors.

1. Introduction

In the operator algebra setting for quantum field theory and quantum statistical mechanics there have been given several examples of von Neumann algebras of types III and II_1, see e.g. [1, 3, 8]. Then one has a von Neumann algebra, a group of unitary operators inducing automorphisms of it, and a unique invariant vector, and one shows the von Neumann algebra is of type II_1 if the invariant vector is a trace vector and type III otherwise. In the present paper we shall prove general theorems roughly to the same effect, and apply them to obtain generalizations of results in the quoted papers and also to describe the representations due to extremal invariant states of asymptotically abelian C^*-algebras.

2. Automorphisms of von Neumann Algebras

Our main results are proved in this section. The proof will be based on the ideas of HUGENHOLTZ [8] together with those of KOVÁCS and SZÜCS [10]. We first recall terminology and results from [10]. Let $\mathscr{R}$ be a von Neumann algebra and G a group of $*$-automorphisms of $\mathscr{R}$. A state ϱ of $\mathscr{R}$ (or more generally, a positive linear map of $\mathscr{R}$ into another von Neumann algebra) is *G-invariant* if $\varrho \circ g = \varrho$ for all $g \in G$. $\mathscr{R}$ is *G-finite* if for each non zero positive operator A in $\mathscr{R}$ there exists a normal G-invariant state ϱ of $\mathscr{R}$ such that $\varrho(A) \neq 0$. Denote by $\operatorname{conv}(g(A) : g \in G)^-$ the weakly closed convex hull of the orbit of A under G. Let $\mathscr{B}$ denote the