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Local Field Equation for A^4-Coupling in Renormalized Perturbation Theory*

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Abstract. For the model of A^4-coupling a finite form of the local field equation is proposed and checked in renormalized perturbation theory.

1. Introduction

As is well known the canonical quantization of the Lagrange density

$$\mathscr{L} = \frac{1}{2}\partial_\mu A_0 \partial^\mu A_0 - \frac{m_0^2}{2} A_0{}^2 - \frac{\lambda_0}{4} A_0{}^4 \tag{1.1}$$

leads to a field equation and commutation relations which are meaningless, at least in perturbation theory. This difficulty is avoided in the abstract formulation of quantum field theory which is based on general principles such as Lorentz invariance, microcausality and spectrum conditions [1]. Indeed, well defined power series can be constructed which solve the basic equations of the theory to all orders [2]. Due to the general nature of the principles the abstract formulation provides a frame for all local and invariant interactions specifying only the number and types of the fields involved and the masses and spins of the stable particles. The question arises how in this framework a specific model can be characterized by imposing a simple and meaningful condition on the field operator.

By analogy to VALATIN's formulation of quantum electrodynamics [3] we propose the field equation

$$\begin{gathered} -(\Box + m^2)\, A(x) = \lambda \lim_{\xi \to 0} j(x, \xi) \\ j(x, \xi) = \frac{:A(x+\xi)\, A(x)\, A(x-\xi): \; - \alpha(\xi)\, A(x)}{g(\xi)} \qquad \xi^2 < 0 \end{gathered} \tag{1.2}$$

as such a condition for the model (1.1) of A^4-coupling. The parameters m and λ denote the physical mass and (suitably defined) coupling constant

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