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Statistical Mechanics of Quantum Spin Systems

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Abstract. The thermodynamic limit of a quantum spin system is considered. It is demonstrated that for a large class of interactions and a wide range of the thermodynamic parameters the equilibrium state of the system is describable by an extremal Z^ν-invariant state (a single phase state) over a C^* algebra of local observables. It is further shown that the equilibrium state may be obtained as the solution of a variational problem involving the mean entropy. These results extend results previously obtained for classical spin systems by GALLAVOTTI, MIRACLE-SOLE and RUELLE.

1. Introduction

In recent articles [1, 2, 3] the statistical mechanics of classical spin systems has been considered and it has been shown that, for a large class of interactions and values of the thermodynamic parameters, the state of equilibrium can be described by an extremal (single phase) Z^ν invariant state over a C^* algebra $\mathfrak{A}$ of local observables. Further it was demonstrated that the equilibrium state may be obtained as the solution of a variational problem involving the mean entropy of the Z^ν invariant states over $\mathfrak{A}$. The purpose of the present article is to derive similar results for a quantum spin system; our methods are those of [2] and [3].

2. Notation

Consider particles on a lattice Z^ν and assume that the occupation number n_i of every lattice point x_i is restricted to take the values $0, 1, \ldots, N$ where $N < +\infty$. We call such a system a spin system; this terminology originates from the fact that $\frac{1}{2}(2n_i - N)$ may be viewed as the value of a spin component.

To describe a quantum spin system we associate with each point $x_i \in Z^\nu$ a Hilbert space $\mathscr{H}_{x_i}$ of dimension $N + 1$ and with the finite set $\Lambda = \{x_1, \ldots, x_v\}$ we associate the direct product space $\mathscr{H}_\Lambda = \prod_{x_i \in \Lambda}^{\otimes} \mathscr{H}_{x_i}$. Further we define the algebra of (strictly) local observables $\mathfrak{A}(\Lambda)$ corresponding to Λ to be given by the algebra $\mathfrak{B}(\mathscr{H}_\Lambda)$ of all bounded