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The Yukawa Coupling of Quantum Fields in Two Dimensions. II*

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Abstract. The Yukawa coupling in two dimensional space time is considered. A space cutoff is introduced in the interaction term V, so that the renormalized Hamiltonian H_{ren} is a rigorously defined bilinear form in the Fock Hilbert space. The main result is that H_{ren} is positive provided the finite part of the renormalization terms are suitably chosen. As a consequence, the Schrödinger equation $(i\partial/\partial t - H_{\text{ren}})\Phi = 0$ can be solved.

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§ 1. Discussion of the Results

We study the energy operator in a model of Quantum Field Theory. The model consists of bosons and fermions interacting with a Yukawa coupling. We consider this model in two dimensional space time and we introduce a space cut-off in the interaction energy V. Thus we write

$$V = \int_{t=0} : \Psi^{\dagger}(x)\, \Psi(x) : \Phi(x)\, h(x)\, dx\,, \tag{1.1}$$

for h a function which is zero when $|x|$ is large. (The limit, $h \to 1$, would remove the space cutoff.) We studied the same problem in [2]; hereafter we refer to this paper as I. We showed that the renormalized total energy operator

$$H_{\text{ren}} = H_0 + V + (\text{infinite counter terms}) \tag{1.2}$$

was rigorously defined as a bilinear form on a domain $\mathscr{D} \times \mathscr{D}$, with $\mathscr{D}$ dense in Fock space. The counter terms depend on two parameters, and these parameters take on infinite values. The parameters can be changed

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