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The Existence of Scalar Lie Fields*

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Abstract. It is shown that the existence of nontrivial scalar Lie fields (i. e. fields whose commutator is linear in the field itself) is not precluded by algebraic consistency arguments. A partial characterization of the simplest algebraic Lie field structures is given. Several examples are presented, one of which may be represented by Hermitian operators in a Hilbert space having a unitary representation of the Poincaré group.

1. Introduction

In discussing the general structure of relativistic quantum field theory it is often useful to have at one's disposal idealized models fulfilling some, though not all, of the usual field-theoretic postulates. One approach to the construction of such models, suggested by O. W. GREENBERG [1], is to consider the possibility of fields satisfying particularly simple commutation relations. A prime candidate is the so-called *Lie field*, for which the commutator is linear in the field itself [1] [2]. In the neutral scalar case this would mean

$$i[A(x), A(y)] = \Delta(x, y) + \int dz\, c(x, y, z)\, A(z)\,, \tag{1.1}$$

where Δ and c are real-valued generalized functions.

The usefulness of a nontrivial Lie field with an asymptotic particle interpretation (assuming such a model exists) is apparent, since the retarded functions can be calculated immediately once one knows the generalized functions Δ and c (ŁOPUSZAŃSKI [2] was the first to call attention to this aspect). Moreover, even without the asymptotic condition, a Lie field theory is *soluble* in the sense that the Wightman functions are uniquely determined by the specification of Δ and c and the assumption of a positive energy spectrum [3].

The initial optimism regarding Lie fields was dampened somewhat by a negative result of D. W. ROBINSON [4]. ROBINSON claimed that the existence of a scalar Lie field which is nondegenerate, in the sense that $[[A(s), A(y)], A(z)]$ does not vanish identically, was precluded by in-

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