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Statistical Mechanics of Lattice Systems

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Abstract. We study the thermodynamic limit for a classical system of particles on a lattice and prove the existence of infinite volume correlation functions for a "large" set of potentials and temperatures.

§ 1. Introduction and Notations

In this article we shall study the statistical mechanics of a classical system on a ν-dimensional lattice Z^ν. We assume that at each lattice point there can be either 0 or 1 particle. We suppose that the particles interact through symmetric translationally invariant many body potentials $\Phi^{(k)}(x_1 \ldots x_k)$. Let $X = \{x_1, \ldots, x_N\}$ be a finite subset of Z^ν, then the potential energy U of N particles located at $x_1, x_2, \ldots, x_N$ is:

$$U(X) = \sum_{k=1}^{\infty} \frac{1}{k!} \sum^{\neq}_{(i_1, \ldots, i_k) \in \{1, \ldots, N\}} \Phi^{(k)}(x_{i_1}, \ldots, x_{i_k}) \tag{1}$$

where $\sum^{\neq}$ extends over all k-ples $i_1, \ldots, i_k$ of distinct indices (between 1 and N); in particular $U(\emptyset) = 0$. We shall consider only interactions $\Phi = (\Phi^{(k)})_{k \geqq 1}$ such that

$$\|\Phi\| = \sum_{k=1}^{\infty} \frac{1}{k!} \sum^{\neq}_{0 \neq x_1 \ldots x_{k-1} \in Z^\nu} |\Phi^{(k)}(0, x_1, \ldots, x_{k-1})| < +\infty \tag{2}$$

where the second sum extends over all $(k-1)$-ples $x_1, \ldots, x_{k-1}$ of distinct lattice points different from the origin 0 of Z^ν. With respect to the norm (2) the set $\mathscr{B}$ of interactions Φ such that $\|\Phi\| < +\infty$ is a (real) Banach space.

§ 2. Definitions and Inequalities

From (1) and (2) we deduce the following stability property:

$$|U(\{x_1, \ldots, x_N\})| \leqq N \|\Phi\| . \tag{3}$$

We define a subspace $\mathscr{B}'$ of $\mathscr{B}$ by

$$\mathscr{B}' = \{\Phi \in \mathscr{B} : \Phi^{(1)} = 0\} .$$

We may write $\Phi = (-\mu, \Phi')$ for every $\Phi \in \mathscr{B}$ with $\mu = -\Phi^{(1)}$ and $\Phi' \in \mathscr{B}$. We interpret μ as chemical potential and denote by U' the potential energy corresponding to Φ'.

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