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# Mean Entropy of States in Classical Statistical Mechanics

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**Abstract.** The equilibrium states for an infinite system of classical mechanics may be represented by states over Abelian $C^*$ algebras. We consider here continuous and lattice systems and define a mean entropy for their states. The properties of this mean entropy are investigated: linearity, upper semi-continuity, integral representations. In the lattice case, it is found that our mean entropy coincides with the KOLMOGOROV-SINAI invariant of ergodic theory.

## 0. Introduction

A new approach to the description of the equilibrium states of statistical mechanics has recently been intensively studied. In this approach these states are identified with states on a $B^*$ algebra $\mathfrak{A}$. It is assumed that the theory is invariant under a group $G$ (for instance the Euclidean or translation group) and the states considered are $G$ invariant. The algebra $\mathfrak{A}$ is Abelian for classical systems and non-Abelian for quantum systems. $G$ invariant states on Abelian $C^*$ algebras may be identified with measures on a compact set which are invariant under a group of homeomorphisms of this set, their study is thus naturally part of ergodic theory. Many of the recent results have consisted in extending ergodic theory to the case of a non-Abelian algebra $\mathfrak{A}$. It would thus be natural to obtain a non-Abelian extension of the mean entropy introduced by KOLMOGOROV and SINAÏ (KOLMOGOROV-SINAÏ invariant). Another reason for doing this is that a mean entropy should, on physical grounds, be associated with the equilibrium states of statistical mechanics (see [10]). In this paper we undertake the more modest project of giving a natural physical definition of mean entropy for classical systems, studying its properties and finding its relations with the KOLMOGOROV-SINAÏ invariant[1].

## 1. States of Classical Statistical Mechanics

The description of equilibrium states in statistical mechanics as states on $B^*$ algebras has been considered recently by several authors

[1] Results similar to those described here have been obtained independently by O. LANFORD (unpublished).