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A Spherically Symmetric Solution of the Maxwell-Einstein Equations

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Abstract. A spherically symmetric solution of the "already unified field theory" of RAINICH (i.e. of the source-free Maxwell-Einstein equations) is presented which represents a static massless charged particle. It is not equivalent to the Reissner-Nordström solution with zero mass, although both metrics repel uncharged test particles.

§ 1. Introduction

In the absence of sources the Maxwell-Einstein equations are[1] (see [2])

$$\left.\begin{aligned} f_{ij;k} + f_{ki;j} + f_{jk;i} &= 0\,, \\ f^{ij}{}_{;i} &= 0\,, \qquad \text{and} \\ R_{ij} - \frac{1}{2} g_{ij} R &= 2 f_{ih} f_j{}^h - \frac{1}{2} g_{ij} (f_{rs} f^{rs})\,, \end{aligned}\right\} \tag{1.1}$$

where the electromagnetic field tensor f_{ij} is defined in terms of the 4-potential A_i by

$$f_{ij} = A_{i;j} - A_{j;i}\,. \tag{1.2}$$

It is well known [2], [3] that the system (1.1) is equivalent to the algebraic conditions

$$\left.\begin{aligned} R_{00} &\geqq 0\,, \\ R &= 0\,, \qquad \text{and} \\ R_{ij} R^j{}_k &= \frac{1}{4} g_{ik} (R_{hj} R^{hj})\,, \end{aligned}\right\} \tag{1.3}$$

thereby giving rise to the so-called "already unified field theory" of RAINICH. In fact, if we are given a metric satisfying (1.3) we can construct the corresponding f_{ij} in the following manner (for details see [3]).

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[1] This note may in some respects be regarded as a continuation of [1] and we shall retain the same notation.