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Space-times Containing Perfect Fluids and Having a Vanishing Conformal Divergence*

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Abstract. The solutions of the Einstein field equations are studied under the assumptions that (1) the source of the gravitational field is a perfect fluid, (2) the divergence of the conformal (Weyl) tensor vanishes, and (3a) either an equation of state exists such that $p = p(w)$, p being the pressure and w the rest energy density, or (3b) the rest particle density is conserved. Under assumptions (1), (2), and (3a) it is shown that the space-time is conformally flat and the metric is a Robertson-Walker metric. The flow is irrotational, shear-free, and geodesic. Under assumptions (1), (2), and (3b) it is shown that either the line element is static or the fluid has a very special caloric equation of state. Conditions for a static solution to exist are examined, and it is shown that the Schwarzschild interior solution satisfies these conditions as does the Einstein universe. The Schwarzschild interior and the Einstein universe are the only conformally flat, static solutions obeying (1), (2), and (3b).

1. Introduction

In this paper we shall discuss the space-times satisfying the Einstein field equations when the source of the gravitational field is a perfect fluid and which are such that the divergence of the conformal (Weyl) tensor vanishes.

It is a consequence of the latter condition that the four-velocity of the fluid has vanishing rotation and shear. In addition, the derivatives of the rest energy density w in directions orthogonal to the four-velocity vanish. If the fluid obeys an equation of state, that is, if the pressure is a function of w alone, it then follows that the particle paths of the fluid are geodesics. Thus the assumptions that the divergence of the conformal tensor vanishes and that an equation of state holds imply that the flow is geodesic, irrotational and shear free [1]. As is known, these conditions in turn imply that the space-time is conformally flat, and that the metric is of the Robertson-Walker type [2].

If the assumption that an equation of state holds is replaced by the assumption that the particle density is conserved, it follows that either the rest energy density of the fluid is constant, or the entropy is constant,

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