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On the Equilibrium States in Quantum Statistical Mechanics

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Abstract. Representations of the C^*-algebra $\mathfrak{A}$ of observables corresponding to thermal equilibrium of a system at given temperature T and chemical potential μ are studied. Both for finite and for infinite systems it is shown that the representation is reducible and that there exists a conjugation in the representation space, which maps the von Neumann algebra spanned by the representative of $\mathfrak{A}$ onto its commutant. This means that there is an equivalent anti-linear representation of $\mathfrak{A}$ in the commutant. The relation of these properties with the Kubo-Martin-Schwinger boundary condition is discussed.

I. Introduction

In statistical mechanics one studies large systems and the aim is to derive the macroscopic, or thermodynamical properties of such systems from the equations of motion of the individual particles. Due to their large size, such systems have features such as phase transitions, transport phenomena, which are absent in small systems. To exhibit such features in their purest form one has to consider the limiting case of infinitely large systems, i.e., systems with infinitely many degrees of freedom. The usual formulation of classical or quantum mechanics and of statistical mechanics does not allow the treatment of systems with infinitely many degrees of freedom. This means that one has to consider large, but finite, systems and take the thermodynamical limit at the end. From a logical point of view it seems advantageous to reformulate the theory in such a way that no recourse has to be taken to finite systems. While this aim has not been reached in a complete and satisfactory manner some interesting partial results are known. Among these we may count the Kubo-Martin-Schwinger boundary condition for "thermodynamic Green's functions" [1] and [2], the properties found by ARAKI and

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