

Analytic Continuation of Group Representations. IV

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Received November 25, 1966

Abstract. The main problem, deforming a subalgebra of a Lie algebra, is treated algebraically, requiring an extensive development of methods of defining multiplications on Lie algebra cohomology cochains. Some applications to differential geometry are also presented.

I. Introduction

As we have already seen [2], one of our main problems can be described in the following way: Suppose \mathfrak{G} and \mathfrak{L} are Lie algebras, with Φ a homomorphism of \mathfrak{G} to \mathfrak{L} . It is possible to “deform” these structures in the sense of defining:

- a. A Lie algebra structure $[X, Y]_\lambda$ on \mathfrak{G} varying with the parameter λ , reducing to the given Lie algebra when $\lambda = 0$.
- b. A one-parameter family of linear maps $\Phi_\lambda: \mathfrak{G} \rightarrow \mathfrak{L}$, each of which is a homomorphism of the $[\ , \]_\lambda$ Lie algebra structure on \mathfrak{G} .

The INONU-WIGNER idea of “contraction” of Lie algebras and representations and the Gell-Mann method of “expanding” representations both suggest that this is the fundamental problem.

In this paper, we shall develop the full algebraic formalism necessary to discuss this deformation problem. As can be seen from Ref. [4], this necessitates studying the “multiplicative” structure on the cochains associated with Lie algebra cohomology. We have delayed presenting this theory because of its complexity, but in this paper we can present a relatively simple independent exposition, and show how it is applied to the interesting deformation problems in a straightforward way. There is considerable overlap in results with work done by A. NIJENHUIS and R. RICHARDSON [5, 6, 9, 10]. However, the methods presented here are perhaps better adapted to the explicit calculations that are necessary to apply the theory to interesting problems of group representations and differential geometry.

It is extremely interesting to notice that our problem (deforming Lie algebras and their representations) and that of K. KODAIRA and D. C. SPENCER [3, 11] on deformation of differential geometric structures

* Work supported by the U.S. Atomic Energy Commission.