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The Center-of-Mass in Einsteins Theory of Gravitation

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Abstract. We prove the existence and uniqueness of a center-of-mass line as well as a center-of-motion line, the latter due to G. DIXON, 1964. The validity of the theorems depends on some assumptions listed in § 2, whose most restrictive ones (in the sense of physics) state a certain weakness of the gravitational field. In the concluding paragraph we give some corrolaries and a very simple application to the problem of motion.

1. Motivation

To determine the motion of a finite number of material particles one needs three sets of equations: (I) the field equations, (II) the equations of motion and (III) the supplementary equations. By (III) we mean all equations (if necessary inequalities too) that determine the problem uniquely, e.g. equations of state. By (II) one usually means a set of differential equations giving as solution timelike curves in the space time V^4 (V^4 meant as solution of (I)), such that the curves are uniquely attached to the particles. Up to now it is unknown whether there exist solutions $g_{ab}(x)$ of (I) such that the support of the matter field $T^{ab}(x)$ is a finite set of timelike lines. We therefore take the point of view that $T^{ab}(x)$ means a collection of *extended* sources (we make it precise in § 2). Now two problems arise: (a) To find a timelike curve uniquely defined by the given matter distribution; (b) EINSTEINS classical problem, to derive from (I) the equation of motion (II) for the curves already found in (a). Obviously (b) makes sense only when (a) has been solved. This paper is devoted to the problem (a).

Up to now an answer to (a) was given either by taking over the center-of-mass line of Special Relativity to curved space time, which works for weak fields in connection with a suitable approximation method (FOCK, 1939, et al.); or by taking singular sources and taking as the required timelike curve the support of $T^{ab}(x)$ (EINSTEIN, INFELD, HOFFMANN[1], 1938; TAUB, 1964; INFELD, PLEBANSKI, 1960 et al.). Taking the

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[1] Here the sources are singular in the mathematical treatment.