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On Local One-Particle Approximations and Locally Conserved Currents

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Abstract. Local one-particle approximations are constructed for matrix elements of two local field operators. If one of the fields is a locally conserved current the approximation is extended in such a way that both locality and current conservation are valid in the approximation.

I. Introduction

The most basic difficulty in any theoretical treatment of elementary particle physics is the fact that we have to deal with an infinite number of intercorrelated functions.

In relativistic quantum field theory such intercorrelations are induced by the infinite set of possible intermediate particle states in matrix elements of field operators. As a first approximation, one can try to take only the discrete one-particle states out of this infinite set and drop all the continuous states as intermediate states. Such an approximation would only be reasonable if all the general properties of the theory are not destroyed by this approximation. In relativistic quantum field theory, it is locality which causes some trouble in this respect.

Because locality is destroyed by the simple one-particle approximation for the commutator matrix element

$$\langle \mathbf{p}|\, [\mathrm{A}_0(x), \mathrm{B}_0(y)]\, |\mathbf{p}\rangle \approx \langle \mathbf{p}|\, \mathrm{A}_0(x)\, |1\rangle \langle 1|\, \mathrm{B}_0(y)\, |\mathbf{p}\rangle - \\ - \langle \mathbf{p}|\, \mathrm{B}_0(y)\, |1\rangle \langle 1|\, \mathrm{A}_0(x)\, |\mathbf{p}\rangle$$

FUBINI and FURLAN [1] got an unwanted $\mathbf{p}$-dependence of the corresponding equal time expression and were forced to take a limit $\mathbf{p} \to \infty$ or $\mathbf{p} \to 0$ to get a consistent result.

In the general frame of relativistic quantum field theory [2], [3] "local one-particle approximations" were first constructed by SYMANZIK [4], [5] for retarded functions and by ZIMMERMANN [6], [7] for time ordered functions. STREATER [8] and STORA [9] have investigated the

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